A novel rapid prototyping system for expandable polystyrene

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Abstract

Purpose – Expanded polystyrene (EPS) is a low-density and cheap material, which has been widely used in commercial areas. As the demand for small-batch, flexible and quick production increases, producing EPS products with metals moulds has become unaffordable. The purpose of this paper is to describe the development of an EPS rapid prototyping (ERP) process, with an electric heating tool.

Design/methodology/approach – Two new cutting strategies for the ERP process, constant angle mode and constant thickness mode, are proposed. The methods to generate tool path of those models are also discussed. In order to improve accuracy and cutting effectiveness, experiments have been carried out to investigate the thermal characteristics in the ERP process. Consequently, the relationships between the size of material removal area and process parameters are obtained. Suitable processing parameters for the ERP system are also conducted.

Findings – It is found that the ERP process can rapidly produce complex three-dimensional parts in one-off clamping without post-processing procedures as in traditional rapid prototyping, such as, extra support removing, step texture finishing and distortion regulating.

Originality/value – The paper provides several examples to explain and illustrate the applicability and workflow of the ERP system.

Keywords Rapid prototypes, Polystyrene, Production equipment, Heat engineering components

Paper type Research paper

1. Introduction

Expandable polystyrene (EPS) is a light, inexpensive and abundant material which is produced as by-product of petroleum. It has been widely applied to package industry, thermal insulation and expandable mould casting. For mass commercial production, EPS products are produced by expanding EPS particles in mould. Metal moulds, which results in large capital, long lead time and poor flexibility of manufacturing are often required.

As requirement of small-batch, flexible and quick production increases, a novel processing method is necessary. The rapid prototyping and manufacturing (RPM) can meet those needs, because of the potential of these technologies to rapidly fabricate three-dimensional parts with geometrical complexity from a CAD model in a CAD/CAM environment (Yan and Gu, 1996). With increasing markets needs, such as advertisements, casting moulds, building decorations and package industry, the RPM technology for EPS materials is also studied and developed to satisfy needs for single or small-bath products in the last ten years.

The RPM process can be classified into two categories for EPS material according to producing method. One is laminated object manufacturing (LOM) using EPS sheet. Horvàth *et al.* (1998) and Broek *et al.* (2002) put forward the idea of free-form thick layered object manufacturing based on a flexible heat blade cutting. Ahn *et al.* (2002) have developed a variable lamination manufacturing method using a four-axis synchronized automatic hot-wire cutter with hot-wire and a parallelogram mechanism. For those laminating processes, post-processing resulting from the nature of layer-by-layer building process is labor-intensive and time-consuming, as it involves the removal of supporters and remaining material after completing parts. The surface quality and dimensional accuracy for that process are also not satisfied. Thus, the improvement of the process is still badly needed.

The other is machining process. Jouaneh *et al.* (1997) have developed a robotic using hot-wire to make simple EPS part. Huang and Lin (2003) developed a dual-robot workcell to shape EPS parts. Hamade *et al.* (2005) employed five-axis robotic arm to move the cutting tool, the tip of which is with a curved hot-wire, into a workpiece (EPS) to form arbitrary shape part. Another process involves making access to a part using an indexing table. Kim *et al.* (2007) devised a three-axis cutter using a hot tool with tangential grooves.

In recent ten years, a serial of CNC apparatus using hot-wire for commercial applications were developed by Croma Co. and Megaplot Co., which can rapidly produce a large-scale two-dimensional EPS part in a few minutes. Most of these works have disadvantages:

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- complexity and high manufacturing costs of apparatuses;
- long time-consuming; and
- low surface quality and dimensional accuracy of complicated-shape EPS products.

Therefore, a new cutting strategy is needed to resolve those problems.

In this material, a new EPS rapid prototyping system, with an electric heating cutter, is proposed. The related apparatus has also been developed to fulfill the EPS rapid prototyping (ERP) process. The methods to generate tool path are studied. Material thermal removing principle of the ERP process is similar to that of other non-contact thermal cutting, discussed by Kim *et al.* (2005), in which unnecessary material is melted and vaporized by heat transferring during the process. Dimensional accuracy and surface quality of a shaped part highly depend on the thermal field distribution in workpiece. Cutting experiments are carried out to investigate the thermal characteristics in the ERP process. Based on the results of experiment, both suitable operating parameters and the relationships between material removal zone and processing parameters are obtained.

2. Introduction of the ERP apparatus

The ERP system can fabricate three-dimensional complex parts in one-off clamping without post-processing procedures as in traditional rapid prototyping, such as, extra support removing, step texture finishing and distortion regulating. So the ERP apparatus can be designed with simple frame mechanism without sticking and bonding components (Figure 1). The apparatus consists of CNC controlled motors at each axis, an electric heating tool, a temperature-control unit of the tool, a feedback unit of position and temperature and controlling software. The cost of ERP apparatus is also quite low.

The specifications of the apparatus are shown in Table I. The processing procedure of the ERP system is provided as follows:

1. Three-dimensional solid modeling of a part is designed by commercial CAD software and saved as an STL file.
2. Cutting model is selected, and then the STL file is sliced into two-dimensional-layer contours. Thermal radius compensation resulting from material removal is applied to those sliced contour data for two-dimensional tool path of each layer.
3. An EPS block is fixed on the apparatus, as shown in Figure 2. Then the tool is heated to a selected temperature above the block evaporated temperature.
4. The control software loads on those path data and sends signal to x, y and α axis CNC controlled motors layer-by-layer to eliminate unnecessary material with the hot tool.

3. Thermal analysis of the ERP process

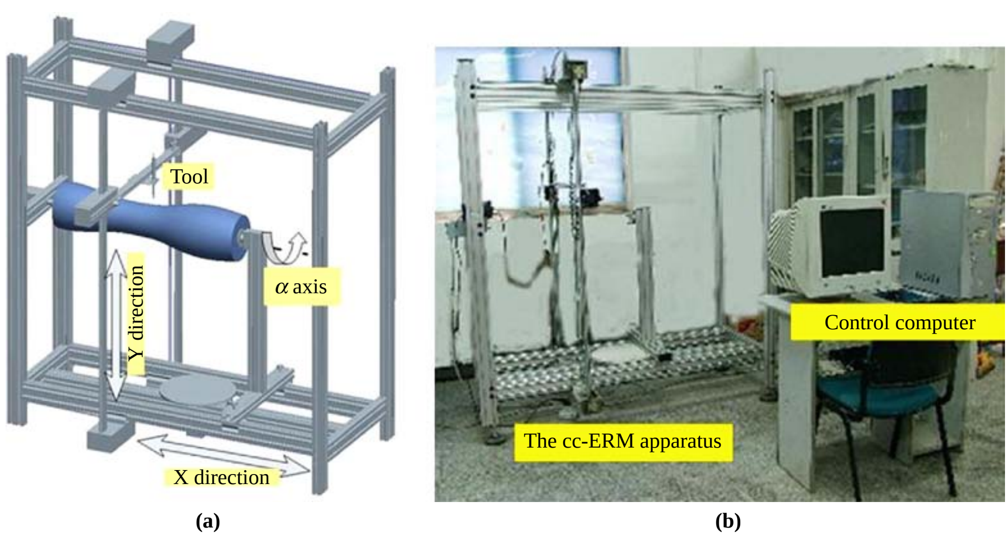
3.1 Experiment parameters

Cutting experiments are carried out on the ERP apparatus by moving the hot tool on samples with one direction. The EPS sample is selected with density of 20 kg/m^3, which is usually applied as pattern of lost-foam casting because of low gas evolution. Major experimental parameters are as follows: the tool diameter is 1 and 1.5 mm. The tool temperature is set to be 200, 250, 300, 350, 400 and 450°C, and moving speed of the tool is 15, 25, 35, 45 and 65 mm/s, respectively.

3.2 Cutting principle of the ERP system

The ERP system is a combination of CNC and LOM and belongs to the material-removing category. The cutting principle is as follows. The hot tool, heated above the decomposition temperature, makes the thermal field of material. According to temperature distribution by the thermal field, material phase is changed at each section. Material close to the tool is melted and vaporized, as shown in Figure 2. Using a hot tool to remove material, the ERP system has several advantages, such as a large material removal rate, a better surface quality and no cutting resistant force at certain temperature and speed range.

Figure 1 Schematics and prototype of the ERP apparatus



Notes: (a) Schematics of the ERP apparatus; (b) prototype of the ERP apparatus

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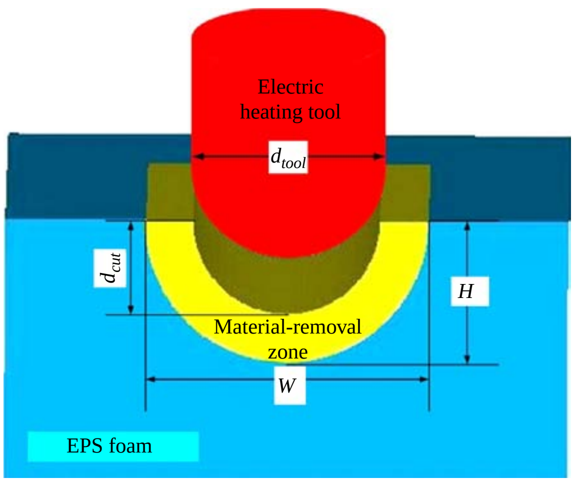
Table I Specifications of the ERP apparatus

Range of working	
Length (*L*)	1,500 mm
Radius (*R*)	300 mm
Angle (*θ*)	0-360°
Electric heating tool	
Diameter	2, 1.5 and 1 mm
Speed	15-65 mm/s
Temperature	200-450°C

Figure 2 Concept of heat cutting using an electric heating tool



3.3 Experiment results and discussion

Experiment results are shown in Figure 3. The width of material-removal zone, namely, kerfwidth, is enlarged as a function of the tool diameter. The difference of the kerfwidth under the same cutting conditions is 0.55 mm and is 1.1 times larger than that of tool diameters. When cutting speed and the tool diameter remain unchanged, the kerfwidth is proportional to the tool temperature and the maximal one is 1.965 mm at 450°C It is almost two times larger than the tool diameter as shown in Figure 3(a). The kerfwidth is also inversely proportional to cutting speed, when the tool temperature remained unchanged. The minimum kerfwidth is approximately equal to the tool diameter 1 mm.

It can be seen that the tool temperature and cutting speed have predominant influence on the kerfwidth of material-removal. The optimum kerfwidth and the tool temperature at a moving speed of 35 mm/s can be derived from the regression with the experimental data shown in Figure 3(a):

$$W = 0.00248^{*}T + 0.557, \quad R^2 = 0.99. \tag{1}$$

The relationship between the kerfwidth and cutting speed at 350°C can also be derived from the regression using those experimental data:

$$W = -0.00966^{*}V + 1.806, \quad R^2 = 0.92. \tag{2}$$

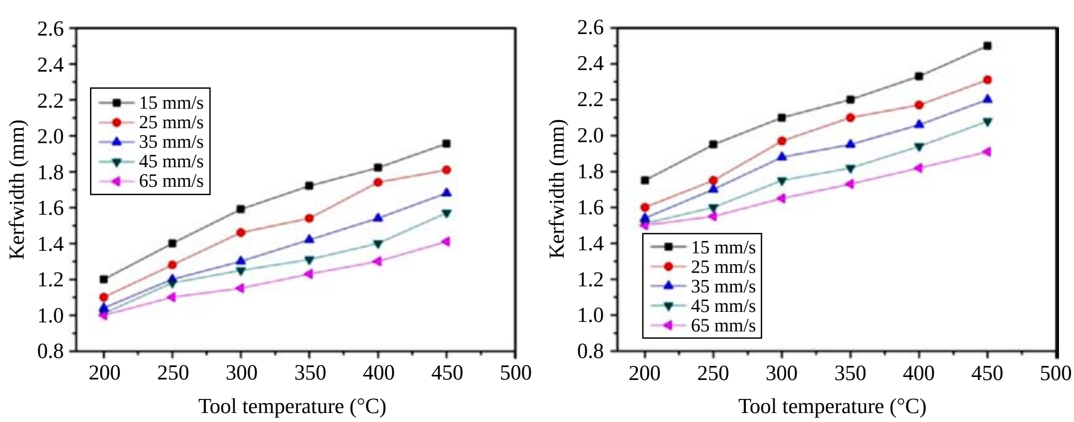
Kim *et al.* (2007) suggested the thermal characteristics in all directions were nearly the same when a heat tool cut EPS block and the cross-section of material-removal zone was a semicircular shape. Thus, the depth of material-removal zone can be described as follows:

$$H = W - d_{tool} + h_{cut}. \tag{3}$$

It is important to maintain stability and small material-removal zone for the surface quality of workpiece. Both faster cutting speed and lower tool temperature can effectively reduce the size of material-removal zone. However, the melted material around a tool cannot be fully vaporized when the tool temperature is lower than the decomposition temperature, and it results in the increase of cutting resistance and the shape of material-removal zone will be rough along the cutting path. On the other hand, high temperature will enlarge material-removal zone and thermal compensation radius, and excessive thermal compensation radius will induce blunt corners in profile. As a result, the accuracy of part will be reduced. Moreover, it has been found experimentally that kerfwidth is stable and smooth, and unwanted material can be fully melted and vaporized along tool path when the kerfwidth is from 1.3 to 1.6 times of the tool diameter. The suitable cutting parameters considering kerfwidth can be devised from equations (1) and (2).

As above discussed, the suitable processing parameters should be as follows: cutting speed between 20 and 50 mm/s,

Figure 3 Relationship between kerfwidth and temperature of tool



Notes: (a) D_{tool} = 1 mm; (b) D_{tool} = 1.5 mm the tool temperature between 300 and 400°C when diameter of the hot tool is 1 mm.

4. Discussion of two novel cutting strategy of the ERP system

4.1 Determination of slicing layer interval

Slicing layer interval, which determines surface roughness and cutting effectiveness, is an important parameter for the ERP system. The surface shape under various slicing layer is sketched in Figure 4. If the interval d_t is larger than the width W of material-removal zone as shown in Figure 4(b), it will result in massive unwanted-material remaining on the part surface after cutting, which causes an increase in the surface roughness. If d_t is less than half of the width W as shown in Figure 4(c), it will produce removal zones overlapped and decrease the cutting effectiveness of the ERP system. Therefore, the slicing layer interval, which is equal to the half width of material-removal zone, is an ideal situation as shown in Figure 4(a). The proper slicing layer in constant thickness mode can be written as follows:

$$d_t = 0.5 \times W. \tag{4}$$

The slicing layer angle in constant angle mode can also be determined as follows:

$$\theta = 0.5 \times W/R_{\max}, \tag{5}$$

where $R_{\max}$ is the maximal radius of a part.

4.2 Comparison of two cutting strategies of the ERP system

The ERP apparatus is similar to a CNC machine from Figure 1 and workpiece can be turned around α-axis during processing. Therefore, two cutting strategies for the ERP system are presented. One is constant angle mode and the other is constant thickness mode, as shown in Figure 5. In constant angle mode, the cutting movement of a layer is divided into moving the tool in X and Y directions. Then workpiece turns a slice angle θ around α-axis for the next layer cutting. Nevertheless, in constant thickness mode, the cutting movement of a layer is completed in two steps:

1 moving the tool in Y direction; and
2 simultaneously rotating block around α-axis.

Then, the tool moves a slice thickness d_t in X direction for cutting the next layer.

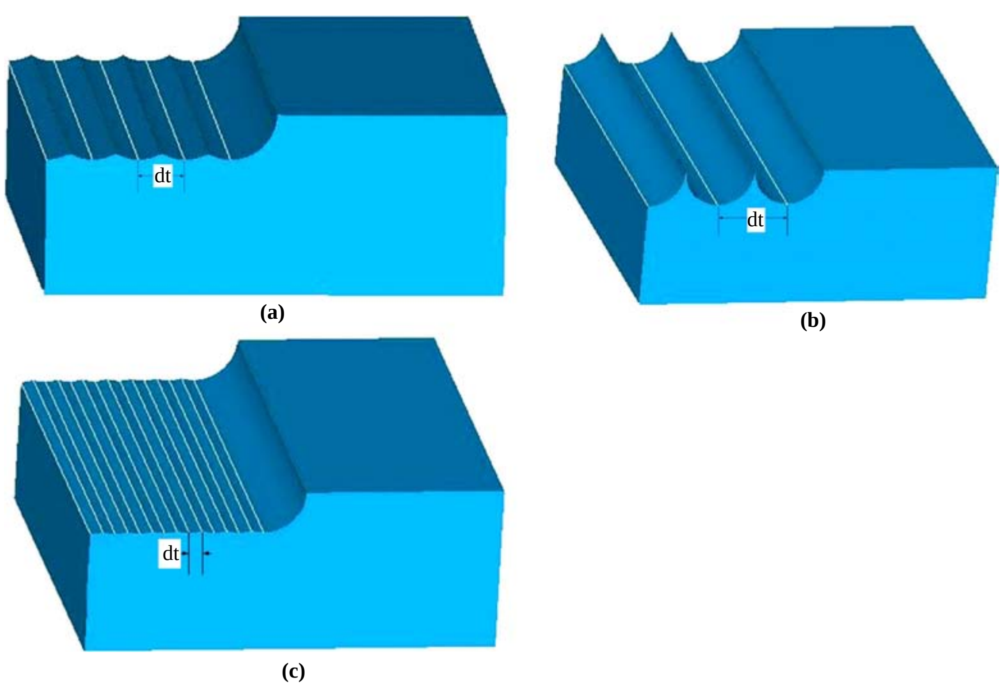
4.2.1 Tool path generation of constant angle mode

Because the constant angle mode is different from conventional RP building method, a new algorithm is needed to generate the path data. The generation procedure of tool path consists of three steps: two-dimensional-layer slicing, layer contour collection and generation of tool paths.

The slicing layer algorithm is summarized as follows: first of all, getting a closed counter polygon by intersection between STL facets and a plane, which is determined by the tool and α-axis. Then, the STL model is turned a layer interval angle θ and a new slice contour can be got with the similar cutting process. Finally, repeating step 2 for $n = 180/\theta$ times, all slice contours are got.

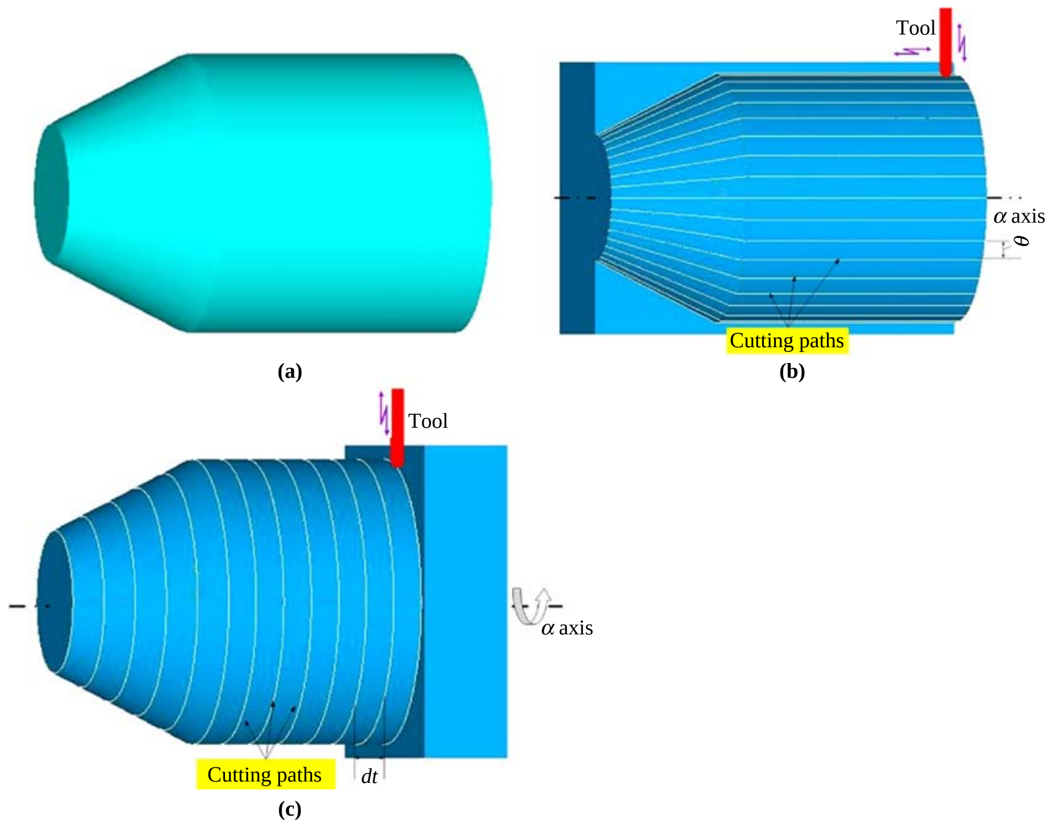
Only part of the slice contour can be cut in one time because the hot tool always lies above the α-axis during the process. Workpiece has to be turned over for cutting the other part of the contour. Obviously, it will cause the reductant work by turning over workpiece frequently. We present a collection strategy of slice contour data to solve this problem, as shown in Figure 6. The collection algorithm can be described as follows: At first, two extreme points P_1 and P_2 in

Figure 4 Surface shape of workpiece under various slicing layer interval



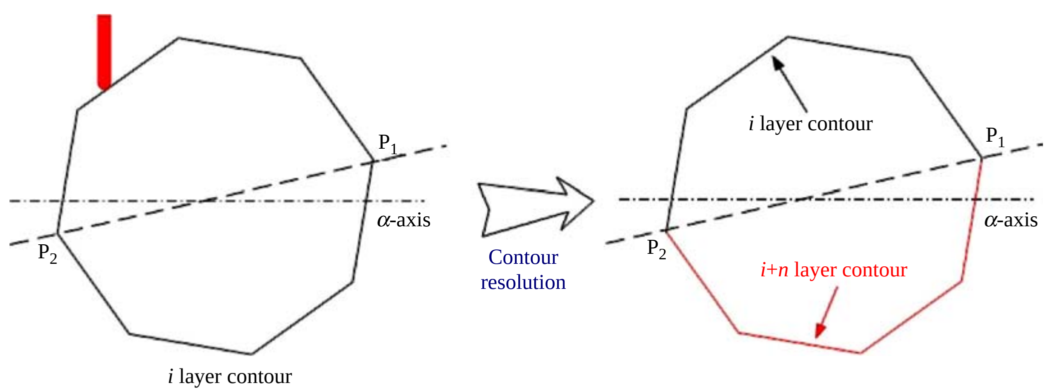
Notes: (a) d_t = 0.5 W; (b) d_t>W; (c) d_t<0.5 W

Figure 5 Cutting strategies of the ERP system



Notes: (a) CAD model; (b) constant angle mode; (c) constant thickness mode

Figure 6 Slice contour treatment in constant angle mode



α-axis direction are taken from the first slice contour data. The point P_1 is set as the starting point and P_2 as the ending point of the contour. Then, the contour data collected from P_1 to P_2 in upward segment are saved as the first layer contour. Next the remained data in this contour in downward segment are saved as the $n+1$ layer contour. Finally, $2n$ layer contours are produced by repeating above steps. Although the number of slice contours is increased by one times after the treatment, processing time is significantly shortened by avoiding turning workpiece frequently.

Thermal radius compensation is added to these layer contours to generate tool path data. The thermal radius compensation is determined as the depth H of material-removal zone according to process parameters.

4.2.2 Tool path generation of constant thickness mode

Generation procedure of tool path in constant thickness mode also consists of three steps: two-dimensional-layer slicing, contour data strategy and generation of tool paths. The two-dimensional-layer slicing algorithm is similar to that of conventional RP technology (Pandey *et al.*, 2003). The slicing layer algorithm is as follows: first, getting a two-dimensional slice contour by intersection between STL facets and a plane, which is perpendicular to α-axis. Then, the plane is moving a layer interval b along the building direction and a new slice contour can be got with the similar intersecting process. Finally, the step 2 is repeated until that all slice contours are finished.

If one directly takes the slice contour data as tool paths, it can result in a prodigious machine error because a layer in constant thickness mode is got by the interpolation movement of α and Y-axis motors controlled by a commercial control card. For example, cutting a line side AB of a square contour, as shown in Figure 7. The movement commands are that motor of α-axis turns 90° and that of Y direction keep motionless with the tool at the location ρ_A above α-axis because of $\Delta\rho = \rho_A - \rho_B = 0$ and $\Delta\theta = \pi/2$. Obviously, the cutting path is an arc rather than a designed line. The error δ, derived in appendix, is inevitable and ubiquity in the coordinated motion of rotary and linear movement, and should be controlled to lesser than a reasonable value to improve the dimensional accuracy in constant thickness mode.

The δ decreases with shortening the line segment length based on geometry. Therefore, the machine error can be decreased by subdividing a long line segment of layer contours into several short pieces using a division algorithm, which is presented by the authors. The division algorithm is summarized as follows:

1 Considering commercial RP with dimensional accuracy of 2 per cent (Muller, 2000), a threshold value T of reasonable cutting tolerance can be calculated by using follow equation:

$$T = 0.01 \times R_{\max} \tag{6}$$

where $R_{\max}$ is the maximal radius of a part.
2 The δ of start line segment of first slice contour is calculated according to appendix. If δ is less than or equal to T, the line segment is kept. Otherwise, it needs to be subdivided, in order to decrease the cutting error.
3 In subdivision procedure, the line segment, first, is subdivided into two new line segments at $\Delta\theta/2$, as shown in Figure 8. Then, comparing the maximal $\delta_{\max}$ of the new line segments with the threshold value T. If the $\delta_{\max}$ is larger than the T, the primitive line segment needs again to be subdivided at $\Delta\theta/3$ and $2\Delta\theta/3$, respectively. Repeating the subdivision procedure until the maximal $\delta_{\max}$ of all line segments are less than or equal to T. Then, these short line segments are replaced for the primitive line segment and saving as contour data.
4 Repeating step 2 and 3 for next line segment of the contour.
5 After finishing one contour treatment, repeating step 2 to 4 for the next contour.

Thermal radius compensation is added to these treated contours data to generate practical tool path data of constant thickness mode.

4.2.3 Difference between two cutting strategies
In general, processing cost of the ERP system is low because of using electric heating cutter instead of laser beam, which is often used in traditional RP system, and EPS material is also inexpensive and abundant. The costs of two cutting strategies are nearly same. However, there are some differences between two cutting strategies, such as, processing time and surface quality.

In constant angle mode, cutting speed is combination of X and Y-axis linear speed and thus, it is roughly constant according to an initial setting during processing. In constant thickness mode, cutting speed is a combination of a-axis tangential speed and Y-axis linear speed. As varying polar radius in layer contour, the cutting speed changes frequently, which causes the average cutting velocity lower than an initial setting value. As a result, machine time of constant angle mode is usually shorter than that of constant thickness mode.

As discussed in Section 4.1, high surface quality and cutting effectiveness of the ERP system can be achieved when the distance of two adjacent cutting paths is equal to the half width of kerfwidth. However, in real cutting process, the distance varies not only with slicing layer interval but also with part shape. The real distance of two adjacent cutting paths is shown in Figure 9. From Figure 9(a), the expression of distance L between two random adjacent cutting paths in constant angle mode can be geometrically obtained as follows:

$$L = \sqrt{(\rho_A - \rho_B)^2 + (2\rho_A \sin\frac{\theta}{2})^2 - 4\rho_A(\rho_A - \rho_B)\sin\frac{\theta}{2}\cos\frac{\pi-\theta}{2}}, \tag{7}$$

where θ is slicing layer angle, ρ_A and ρ_B polar radiuses of point A and B, respectively. Similarly, the expression of distance L in constant thickness mode can also be geometrically obtained from Figure 9(b) as follows:

Figure 7 Schematic of cutting error in constant thickness mode

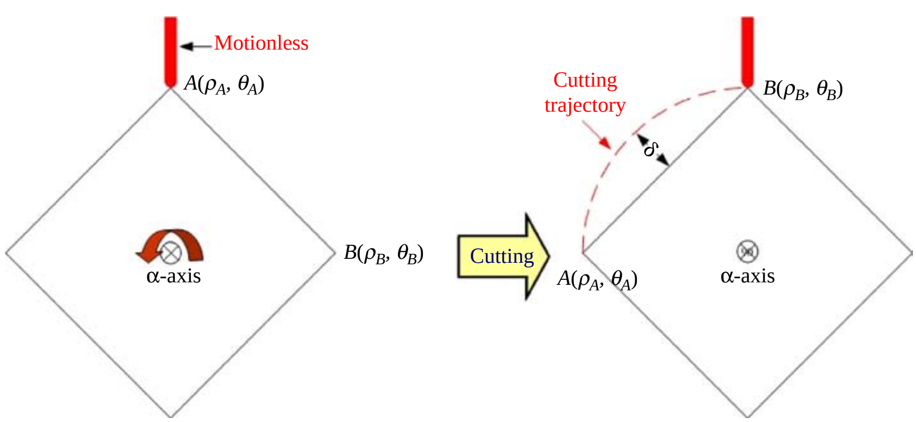


Figure 8 Slice contour treatment in constant thickness mode

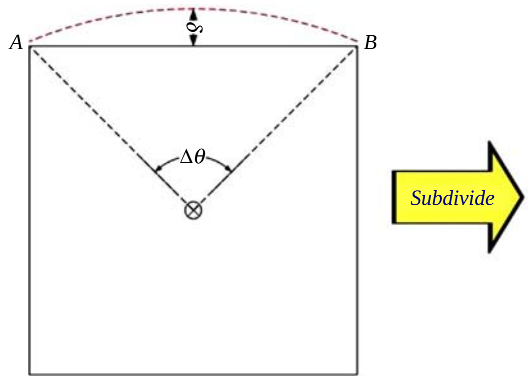
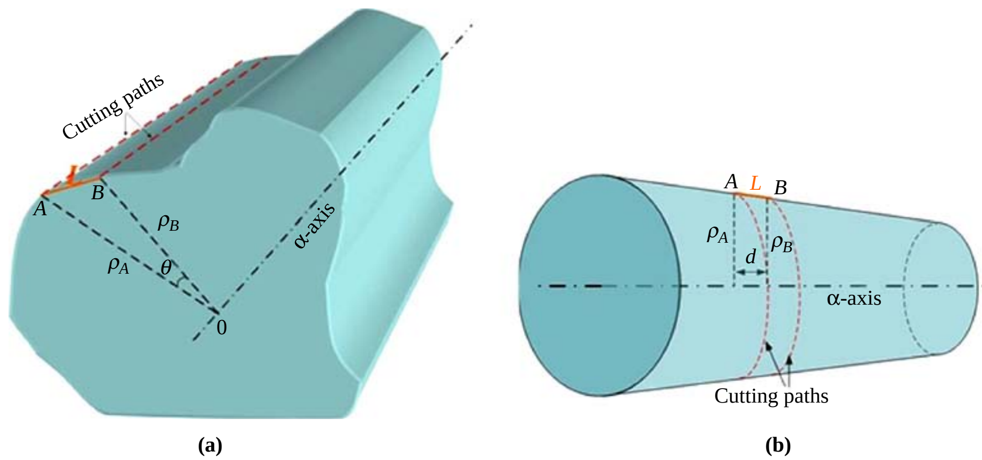


Figure 9 Schematic of the real distance of two adjacent cutting paths



Notes: (a) Constant angle mode; (b) constant thickness mode

$$L = \sqrt{d^2 + (\rho_A - \rho_B)^2}, \quad (8)$$

where d is slicing layer thickness. It is noteworthy that the polar radius difference, $(\rho_A - \rho_B)$, in equations (7) and (8) is a small value when the slicing layer interval is reasonable. Furthermore, it can be found that the distance L in constant angle mode is mainly related to slicing layer angle and polar radius from equation (7), and the distance L in constant thickness mode mainly depends on slicing layer thickness from equation (8). Thus, as polar radius of part varies largely, a larger fluctuation of distance L can be noticed in constant angle mode, and the surface quality is often lower than that in constant thickness mode.

An experimental mode with change rate of polar radius 1.73 and 1.0 on two sides along a-axis is taken as comparison between constant angle mode and constant thickness mode, as shown in Figure 10(a). Parts in Figure 10(b) and (c) are fabricated using constant angle mode and constant thickness mode, respectively. Setting maximum cutting speed as 50 mm/min and tool temperature as 350°C, the processing times for constant angle mode and constant thickness mode are 3 min 35 s and 3 min 53 s, respectively. It can be found that part surface quality in Figure 10(c) is apparently better than that in Figure 10(b).

In short, constant angle mode is more suitable for part with little change rate of polar radius and requirement on short cutting time, while constant thickness mode does well in fabricating part with large polar radius variation and high demand in surface quality.

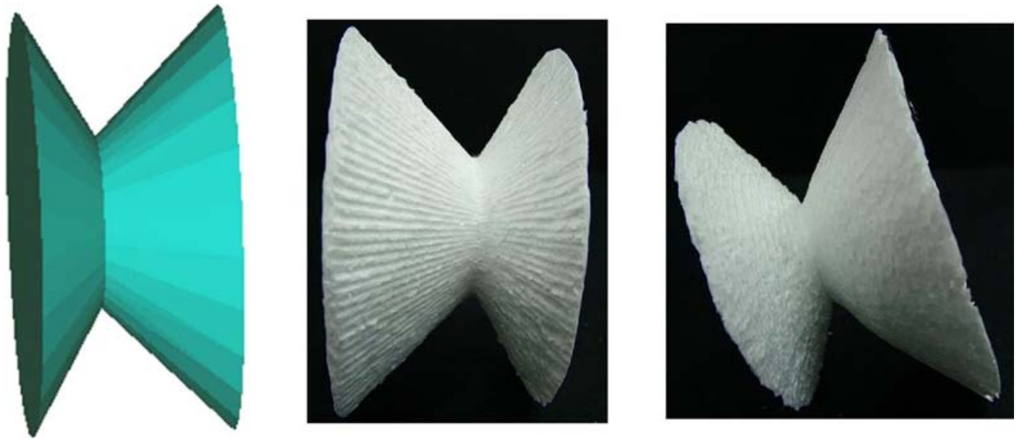
4.3 Three-dimensional practicality mode of the ERP system

To examine the dimensional accuracy, applicability and the effectiveness of the ERP system, several three-dimensional parts have been fabricated. Table II gives the working conditions.

4.3.1 Hemisphere test model

The hemisphere test model has a continuous shape change from bottom to the top, so the model is a basic example to test the geometric suitability and the dimensional accuracy of the ERP system. The size is $100 \times 100 \times 50\,\text{mm}^3$. When cutting depth equals the tool radius, the width and depth of material-removal zone are nearly the same according to equation (3). The thickness of a slice layer is 0.8 mm, which is calculated by equations (1) and (4). Thermal radius compensation value is 1.5 mm, calculated by equation (3). The CAD model and the processing model are shown in Figure 11. As it shows, the final shape has a smooth counter along cutting direction. Fabricating the hemispherical part takes approximately ten minutes. The sizes of part $99.9 \times 99.3 \times 49.5\,\text{mm}^3$ are

Figure 10 CAD model and prototype of experiment part with two cutting strategies



(a) (b) (c)

Notes: (a) CAD model; (b) constant angle mode; (c) constant thickness mode

Table II Fabrication conditions of the ERP system

Density of EPS foam (kg/m^3)	Cutting speed (mm/s)	Tool temperature (°C)	Tool diameter (mm)
20	30	350	1

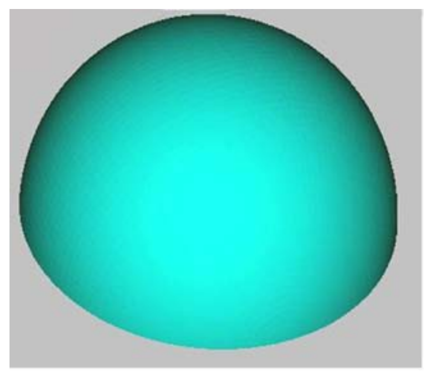
measured using CPJ-3000CZ non-contacting measuring projector. Considering commercial RP with dimensional accuracy of 2 per cent (Muller, 2000), the ERP system can satisfy the requirement of dimensional accuracy.

4.3.2 Cutting examples

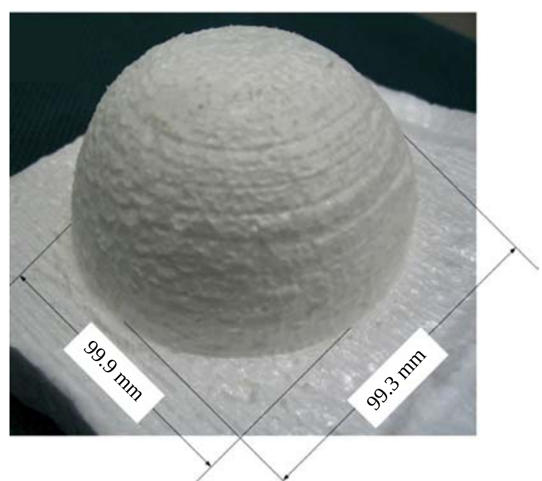
The human head model is as the first cutting example to examine the present ERP system. The human head model contains all the geometrical characteristics for the general three-dimensional shapes, such as, convex, concave shapes and some sudden geometrical change. The cutting procedure of the ERP system is as follows. At the first stage, after the CAD data are loaded onto the CAD/CAM software for the ERP process, process parameters of cutting paths are generated accordingly to the cutting mode. The cutting paths are loaded on control software and the workpiece is set up on the ERP apparatus along α-axis. After the origin of workpiece and tool should be coincided, a cutting process is performed to create the part. The whole process from CAD model to three-dimensional prototypes is shown in Figure 12. The maximal contour size of human head is $158 \times 258 \times 287\,\text{mm}^3$. In a constant angle mode, the slice angle interval is 0.2°, determined by equation (5). In a constant thickness mode, the slice thickness is calculated using equation (4) and the value is 0.8 mm. All shapes have been built within few hours. It can be seen from Figure 12 that three-dimensional model matches CAD model well.

The other examples are a soft drink bottle and a Roman column model which can be easily fabricated by the present ERP apparatus applied in advertisements and building decorations. The maximal sizes of bottle and Roman column are $70 \times 70 \times 210\,\text{mm}^3$ and $200 \times 200 \times 500\,\text{mm}^3$, respectively. As shown in Figures 13 and 14, the final EPS prototypes illustrate fairly satisfied shape compared to original

Figure 11 CAD model and prototype of a hemispherical test model



(a)

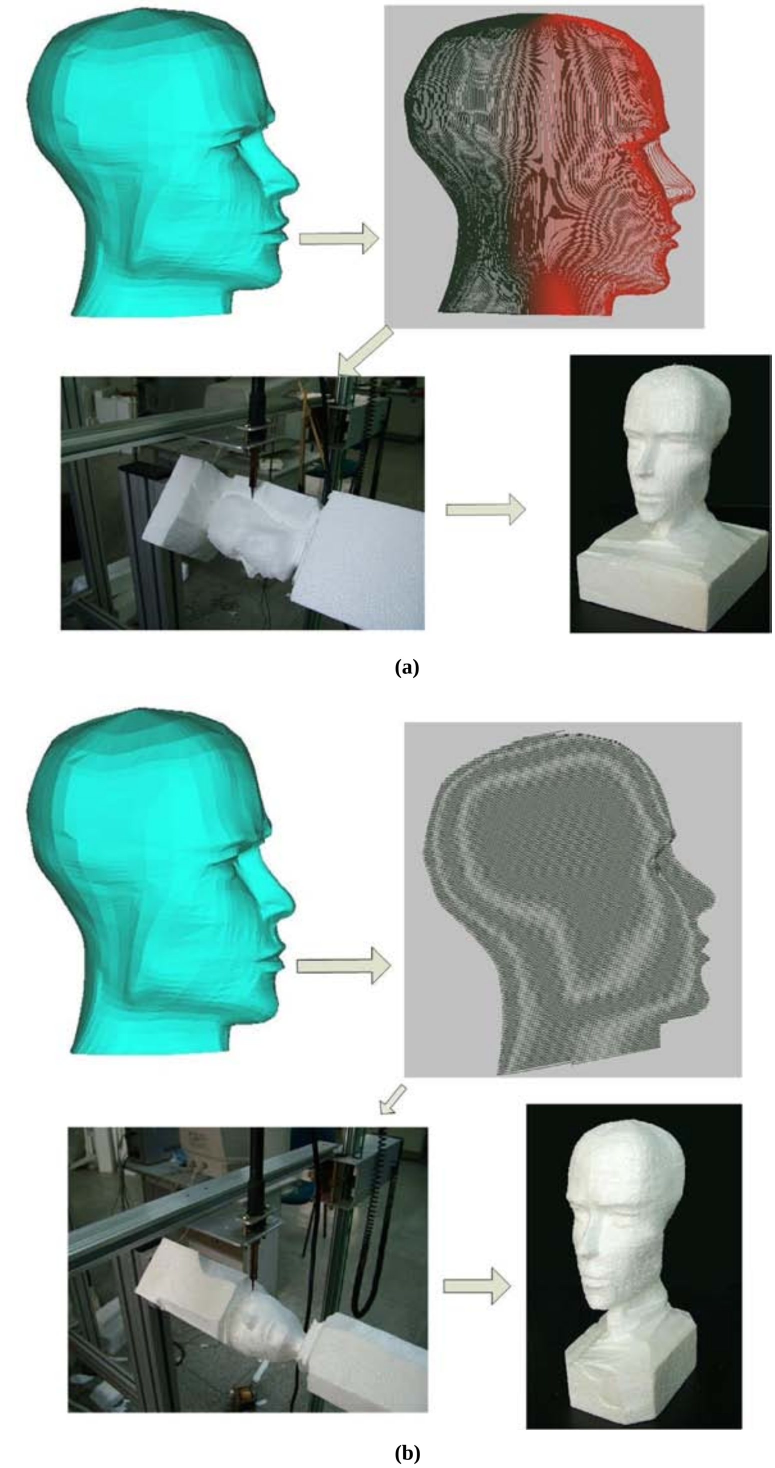


(b)

Notes: (a) CAD model; (b) prototype

CAD models. It can be concluded that the present ERP system is successful to construct three-dimensional workpieces to satisfy the increasing requirements of advertisement, civil building decoration and other application parts.

Figure 12 Process flow of a human head model with vary cutting strategy



Notes: (a) Constant angle mode; (b) constant thickness mode

5. Conclusions

In this paper, a new RP process (ERP) which combines CNC and LOM, is developed for EPS products. Two cutting strategies, constant angle mode and constant thickness mode, are applied for the ERP system. The ERP system can rapidly produce a complex shape part in only one step without requiring post-processing procedures such as extra support removing, step texture finishing and distortion regulating.

As a comparison between constant angle mode and constant thickness mode, it can be found that constant angle mode is more suitable for part with little change rate of polar radius and requirement on short cutting time, while constant thickness

Figure 13 CAD model and prototype of a bottle model

(a) (b)

Notes: (a) CAD model; (b) prototype

Figure 14 CAD model and prototype of a Roman column model

(a) (b)

Notes: (a) CAD model; (b) prototype

mode does well in fabricating part with large polar radius variation and high demand in surface quality.

It is also found that cutting speed and the tool temperature are major parameters for determining the size of the material removal zone according to cutting experiment results. Suitable process parameters for the ERP system should be a cutting speed from 20 to 50 mm/s, and the tool temperature between 300 and 400°C when the tool diameter is in 1 mm.

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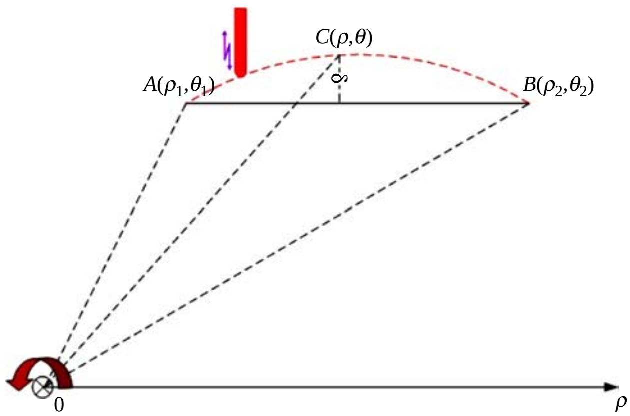
Appendix

In general, a line AB of slice contour can be rotating around α-axis until it is perpendicular to the π plane and above α-axis as shown in Figure A1. The polar coordinates of points A and B can be written as follows:

$$\begin{aligned} \rho_A &= \rho_1, \theta_A = \theta_1, \quad \text{for point A} \\ \rho_B &= \rho_2, \theta_B = \theta_2, \quad \text{for point B} \end{aligned} \tag{A1}$$

Due to rates of two axis keep constant during coordinated motion, respectively, the polar radius and angle of trajectory

Figure A1 Cutting error of a contour line in constant thickness model



point of cutting can be written as follows:

$$\rho = \rho_1 + V_\rho t, \text{ and} \quad \theta = \theta_1 + \omega t, \tag{A2}$$

where t is time. When the cutting finish, namely, the tool moves to point B, equation (A2) can be rewritten as follows:

$$\frac{V_p}{\omega} = \frac{\rho_2 - \rho_1}{\theta_2 - \theta_1} = \frac{\Delta\rho}{\Delta\theta} \tag{A3}$$

From equations (A2) and (A3), the cutting trajectory can be described as follows:

$$\rho = \rho_1 + \frac{\Delta\rho}{\Delta\theta}(\theta_1 - \theta). \tag{A4}$$

Also, the line AB can be described as follows:

$$\rho \sin\theta = \rho_1 \sin\theta_1 = \rho_2 \sin\theta_1 \tag{A5}$$

So the distance of a point at cutting trajectory to line AB can be derived from equations (A4) and (A5) as follows:

$$\delta = \frac{\Delta\rho}{\Delta\theta}(\theta_1 - \theta)\sin\theta + \rho_1(\sin\theta - \sin\theta_1) \tag{A6}$$

To operate the first derivative of δ with respect to θ and let it equal to zero, polar angle of a point, which has the maximal error δ_{max}, can be obtained from equation (A6) as follows:

$$\tan\theta + \theta = \theta_1 + \frac{\Delta\rho}{\Delta}\rho_1 \tag{A.7}$$

The value of θ is got from equation (A7) using numerical calculated method. It is substituted to equation (A6) and the maximal cutting error δ_{max} of line AB is obtained.

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