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ORIGINAL PAPER

Variance of Particle Size: Another Monitor to Evaluate Abrasive Wear

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Abstract The effect of variance of particle size plays a dominant role in the wear rate of materials. In the present research, such effect is investigated using computer simulation, in which Monte Carlo method has been combined to calculate the wear rate of the materials. Abrasive wear rates of materials with the same particle average size exhibited a rapid increase with increasing variance of particle sizes distribution from small variance. Then wear rates maximize and decrease slightly with further increase of variance. The relationship between particle variances and hardness is also investigated. The particle size effect is explained via comparison of wear pattern of different particle sizes.

Keywords Variance of particle size · Abrasive wear · Particle size effect · f_{ab} factor

1 Introduction

Abrasive wear is a stochastic process because of abrasive particles with statistical size distribution. Therefore, the worn surfaces also have strong stochastic characteristics. The key to study abrasive wear is the ability to precisely qualify such worn surfaces. However, it is still difficult to numerically evaluate the worn surfaces using classical mathematical models [1–5]. To this end, Monte Carlo simulation opens a new avenue for qualitatively studying the stochastic process of abrasive wear, providing good evaluation of the influence of particle size and material hardness during abrasive wear [6, 7].

On the other hand, it is reported that for small particles the wear rate increases with increasing particle size [8]. Above the critical size, the wear rate becomes almost independent on further size increasing [9]. Several assumptions have been presented to explain the size effect [10–18]. For example, Borruto et al. [19] showed that the size effect was caused by the differences in wear resistance of the surface layers, that is, small particles mainly penetrate into the surface layer, while large particles can penetrate through it and into the bulk. Additionally, the reduction of the load caused by clog of wear debris can also lead to a size effect [20]. Lasen-Badse [21, 22] suggested that the elastic contact of some small abrasive particles would not contribute to the material removal. Jiang's work established that material removal fraction is the key to understand size effect [10]. Moreover, Aifantis [23] suggested a material strengthening mechanism, often called the "strain gradient effect," at reduced scales. Clearly, among various analysis of size effect, the understanding of the particle size effect is still a matter of debate [4].

Notably, two major factors controlling the abrasivity of a particle are its size, which is usually defined by the expectation, and the variance of particle size distribution. However, as mentioned above, all the studies on the "size effect" merely focused on the average size of particles (i.e., the expectation of particle size), no one has considered the influence of the variance of particle size distribution on wear process. One reason is that, in experiments, particle size can be controlled easily by screening. By contrast, to selectively control the variance of particle size is much

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more difficult. However, the wear rate of material surface will be predominantly determined by the very small amount of the larger particles mixing with smaller particle. Consequently, neither large particles nor small particles with uniform size will induce severe wear because the applied force will be evenly and uniformly distributed to each particles during abrasive wear process. Thus, in our viewpoint, without considering the variance of grit size distribution, it is impossible to give a satisfactory understanding to the effect of abrasive particle size on wear.

Herein, we presented the study of the influence of variance of abrasive particle size by Monte Carlo simulation of two-body abrasion process. Interestingly, we found that the variance of particle size, instead of the average size of particles (i.e., the expectation of particle size), played a crucial role in the "size effect" in abrasive wear. Furthermore, a rapid increase of wear rate was noticed with increasing variance of particle size when the variance value is relatively small. Moreover, for abrasive particles with large variance of particle size, the wear rates are almost the same, which are independent of the average particle sizes. This is because abrasive wear rate is predominantly determined by the small portion of large particles among all the abrasive particles, rather than the majority of the abrasive particles. Additionally, we experimentally checked variances of abrasive particles of different grit number and found similar relationship between the wear rate and variance of abrasive particle sizes. Our results showed that variance of abrasive particle size should be an important parameter to be considered in abrasive wear, providing useful insights in practical applications of abrasive wear.

2 Modeling Procedures of Two-Body Abrasion

Before starting the simulation, a 200 mm × 20 mm abrasive belt is produced by Monte Carlo method based on previous work (Fig. 1a) [6, 7]. In Monte Carlo method, abrasive particle size follows Gaussian distribution and abrasive particle position follows uniform distribution, separately. The abrasive particles are randomly and evenly "planted" on the surface one by one by computing algorithm, from which concentration of abrasives per unit area could be determined. Then, the abrasive paper was pressed onto the worn surface with constant applied force, thereby some of the abrasive particles penetrate into the worn surface. Under the applied force, the abrasive paper slides along the defined direction. One slide over the entire worn surface was defined as one step in simulation. Consequently, the morphology of the worn surface changed from step to step (Fig. 1d). Notably, in this work, revolution parabolic particle geometry is applied for constructing the shape of tips in the simulation (Fig. 1c) [7]. The load acting on each particle is calculated by the contact model we proposed in our previous work [7].

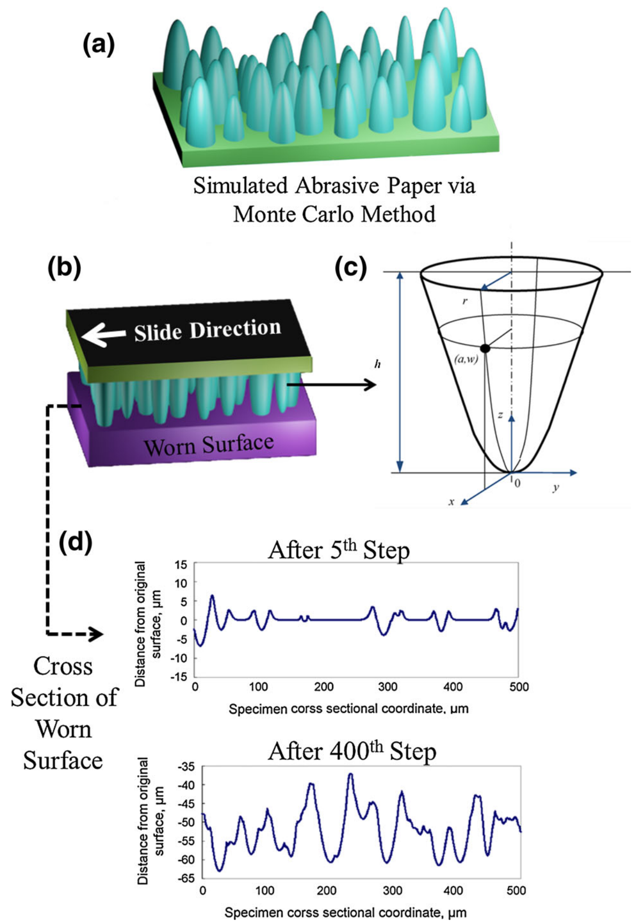

Fig. 1 Schematic illustration of the Monte Carlo simulation procedures of abrasive wear. **a** A small zone of abrasive paper generated with random distributed abrasive particle. Both the height and radius of the abrasive particle follows Gaussian distribution. **b** Sliding the abrasive paper over the worn surface. **c** Abrasive particle in parabolic shape. **d** Partial cross-sectional view of the simulated worn surfaces

The sliding direction length is defined as 200 mm in this simulation. Mesh subdivision size is determined by average abrasive particle diameter. In the grooving direction, the mesh size is generally 1/10 of particle size. In the direction of vertical grooving direction, the mesh size is chosen as 1/1,000 of particle size. The simulated step size is 1×10^{-6} m. The simulated specimen is 4 mm × 4 mm in size. The simulated wear data can be easily transformed to match with the experimental data with respect to equivalent normal pressure.

The linear wear rate of all the mesh units after the Nth step's sliding can be calculated by

$$\varepsilon_{\mathrm{L}} = \frac{z_0 - n_x^{-1} n_y^{-1} \sum_{i=1}^{n_x} \sum_{j=1}^{n_y} z_{i,j}}{N l_{\mathrm{step}}} \quad (1)$$

where z_0 represents the average height before the first sliding step and $z_{i,j}$ is the z coordinate values for the mesh unit $[i, j]$ at the Nth sliding step. Here, n_x and n_y are, individually, the subdivided point numbers in x and

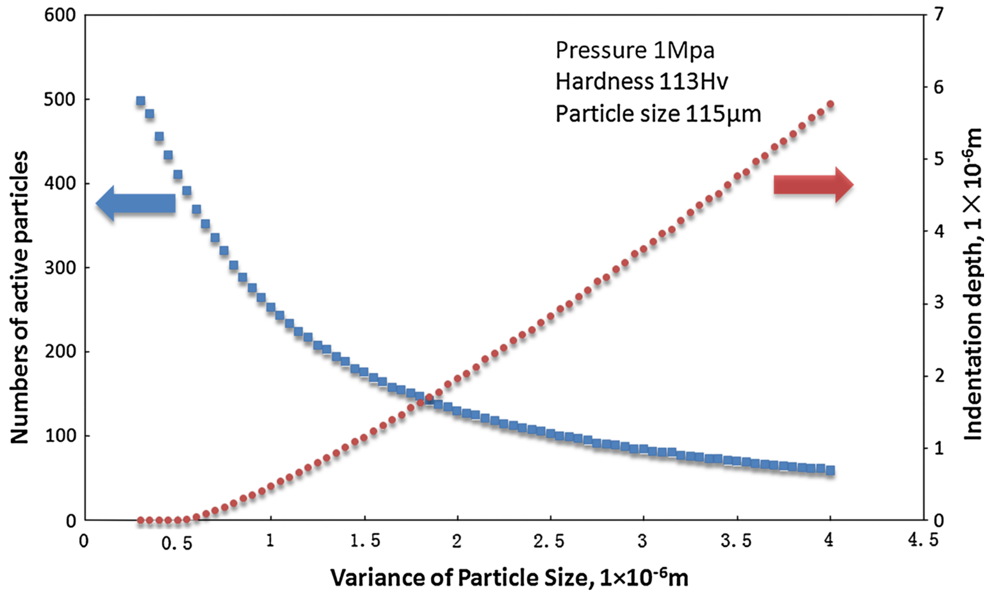

Fig. 2 Numbers of active particles and average indented depth of particles in different variances

y directions. l_{step} is the length of per sliding step. All z values in the specimen surface, therefore, define the morphology of worn surface after the Nth sliding step. The volume wear rate of the specimen can also be determined by multiplying the worn area by ε_L.

3 Analysis of Variance of Particle Size

The active particles are the particles which touched and plowed the worn surface during abrasive wear. As shown in Fig. 2, the number of active particles decreased with increasing the variance, the reason of which is straightforward: larger variance of particle size resulted in a smaller portion of abrasive particles being contact with the worn surface. On the other hand, as the number of active particles decreased, the load applied on each active particles increased, thereby leading to increase in the average indentation depth.

Considering contact of a single particle, the fully plastic contact load P and wear of material per meter caused by a single particle can be given as [6, 7]

$$P = \frac{4}{3}\pi w^{3/2}\left(\frac{r^2}{h}\right)^{1/2} H \tag{2}$$

$$W = \frac{4}{3} w^{3/2}\left(\frac{r^2}{h}\right)^{1/2} f_{ab} \tag{3}$$

$$f_{ab} = \frac{A_g - (A_a + A_b)}{A_g} \tag{4}$$

where w refers to indentation depth, r and h are the particle contour parameters of rotation paraboloid, as shown in Fig. 1c, H is the hardness of material, f_{ab} is a factor exhibiting material loss rate in sliding, A_g is the area of groove, and A_a and A_b are the area of ridges on the two side of sliding groove [24, 25]. With a constant load P, wear rate of a single abrasive particle in Eq. (3) can be expressed as

$$W = \frac{P}{\pi H} f_{ab} \tag{5}$$

As shown in Eq. (5), the factor f_{ab} plays an important role in abrasive wear. The relationship between the factor f_{ab} and indentation depth is given in Fig. 3. It can be noticed that although the number of active particle decreased as the variance of particle size increased, indentation depth also increased at the same time (shown in Fig. 2). Simultaneously, the increase in indentation depth leaded to a rapid increase of the factor f_{ab} (shown in Fig. 3), thereby increasing the wear rate of each individual abrasive particles during abrasive wear according to Eq. (5). To this end, the relationship between the variance of particle size and over all wear rates cannot be determined directly by the wear model of a single abrasive particle. Such difficulty in predicting overall wear rates of an abrasive paper from a single abrasive particle exists in almost all the mathematical models. However, with the analysis of the wear rate of a single abrasive particle, we can be easily calculated the overall wear rate via Monte Carlo simulation, which will be discussed in the following sections.

4 Results and Discussions

4.1 Relation of Particle Size Variance to Two-Body Abrasive Wear Rate

According to abrasive surface with asperities of Gaussian distribution, the total wear rate can be expressed as from Eq. (5):

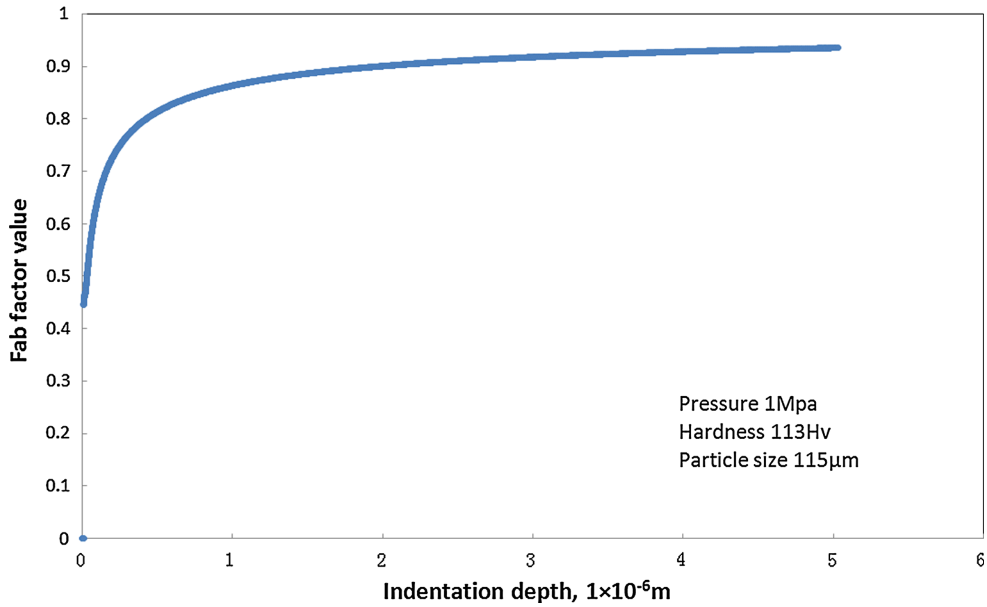

Fig. 3 Finite element method (FEM) simulation results of relationship between the factor f_{ab} and average indentation depth

$$W = W_i = \frac{1}{\pi H}\sum_{i=0}^{n}(P_i \times f_{ab}^{i}) \tag{6}$$

$$P = \sum_{i=0}^{n} P_i \tag{7}$$

where n is the sum of all asperities and W_i, P_i and f_{ab}^{i} present the wear rate, load and the factor f_{ab} of a single particle, respectively. As shown in Fig. 3, the factor f_{ab} tends to be a constant which is about 0.9 when indentation depth is larger than 3 μm. Thus, Eq. (6) can be simplified as:

$$W < \frac{1}{\pi H}\sum_{i=0}^{n}(P_i \times 0.9) = \frac{0.9}{\pi H}\sum_{i=0}^{n}(P_i) = \frac{0.9P}{\pi H} \tag{8}$$

Computer simulated results of wear rates in two-body abrasion are shown in Fig. 4. The related particle size is 115 μm, and pressure is defined 1 MPa. The scattered small points of Fig. 4 are stochastic simulated data (150 points for each variance) with the same variance, and the bigger dots present the average value of wear rates. The curve is divided into two zones by dashed line. In the first zone with remarkably low variance, particle size distribution will be restricted in a quite small range, that is, particles are almost of the same height. Hence, the force applied on wear surface was undertaken by almost every particle. Both the force and the factor f_{ab} on each individual particle were small, which can hardly result in effective wear. Additionally, particle

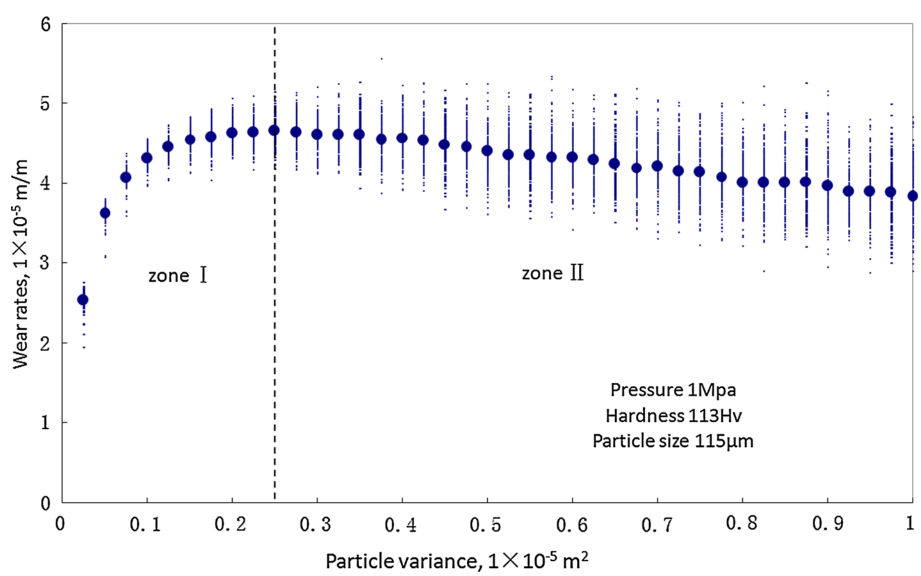

Fig. 4 Relationship between particle variance and wear rates

variances in such zone are smaller than 2.5×10^{-6} as shown in Fig. 4, f_{ab} for most asperities do not reach a threshold value in Fig. 3. Wear rate showed a remarkable increase with variance referred to Eq. (6) and Fig. 3.

In the second zone whose particle variances are larger than 2.5×10^{-6}, wear rate slightly decreased with variance. The reason is that larger particles dominate whole wear process, and f_{ab} tends to be a constant 0.9. Therefore, wear rate keep steady after that threshold value, about $0.9P/\pi H$ according to Eq. (8).

Therefore, it can be seen that there existed an optimal value of particle variance. Such value could be a key parameter to give a strong influence on the wear rate of materials. Influences of material hardness, average particle size and the particle variance on wear rate are discussed in following sections.

4.2 Wear Rate of Materials Relative to Hardness and Particle Variance

The effect of material hardness as well as particle variance to wear rate is exhibited in Fig. 5, in which the average wear rates of different hardness materials are presented in bigger dots. The five biggest dots in Fig. 5 refer to "feature variances" for five different material hardnesses, at which the wear rates of the materials reach to the summit. The feature variance rises with the increase of material hardness, and the average wear rate decreases with increasing material hardness on the contrary.

Figure 6 shows curves of active particle numbers relative to material hardness and particle variance. Active particle numbers in Fig. 6 for high hardness material decreased more slowly than that for low hardness material as particle variance increased. The curve gradients for a sharp decrease in Fig. 6 for different hardness material can be attributed to the feature variance. Above results are beneficial to choosing appropriate abrasive size range in connection with different hardness material in engineering applications.

4.3 Wear Rate Relative to Particle Sizes

The size effect is well known in two-body abrasion. In short, the size effect denotes that for small particles the wear rate increases with increasing particle size. Above some critical particle size, the wear rate becomes almost independent of any further size increases. This critical size is often about 100 μm [9].

However, the difference in wear rates of particle sizes range from 45 to 115 μm is not very significant in case of that particle variances are the same, as shown in Fig. 7. As illustrated, small particle sizes cannot always lead to lower wear rate. Small particles with larger variances can also cause high wear rates. The influence of particle variance on wear rate of material is more distinct than particle size in this case. The increase of variance in a defined average particle size means the increase of size distribution extent. In the meantime, large size particle numbers are also increased. Those large size particles will definitely cause more serious wear. During analyzing the influence of particle sizes on wear, the particle variance cannot be overlooked. Considering grounding process in mechanical workshop as an example, a reasonable abrasive size distribution will improve grounding efficiency of machine besides particle size expectation.

Meanwhile, the variances relative to particle sizes for different abrasive papers are examined using laser scanning confocal microscope (see Table 1). Graphic explanations

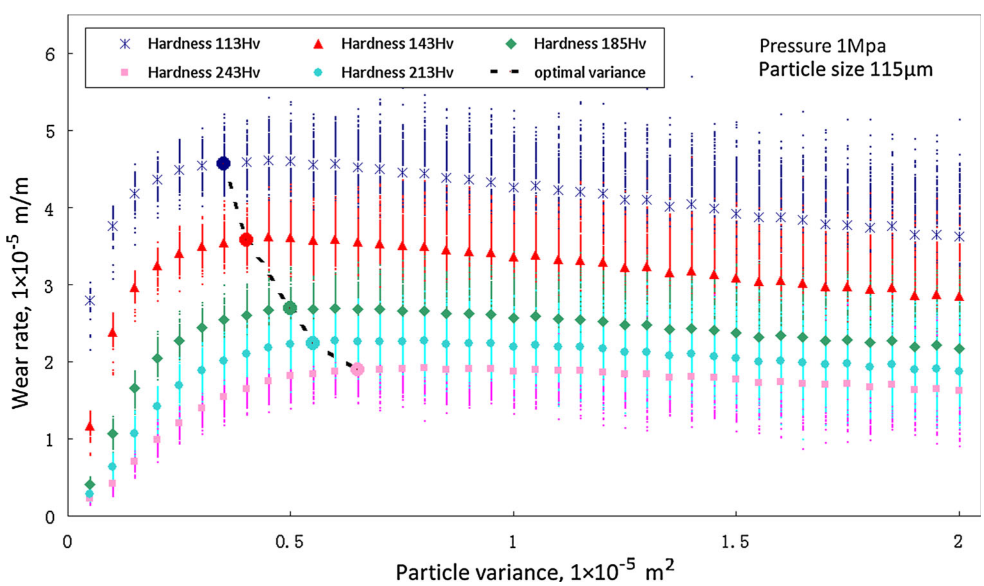

Fig. 5 Wear rates of materials with different hardness and variance

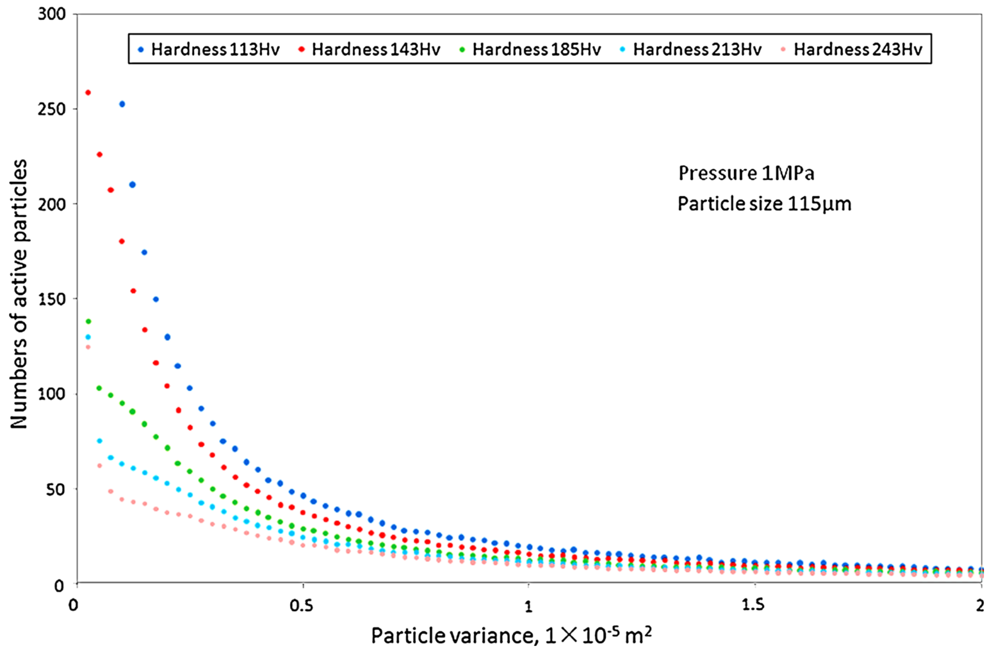

Fig. 6 Active particle numbers with different hardness and variance

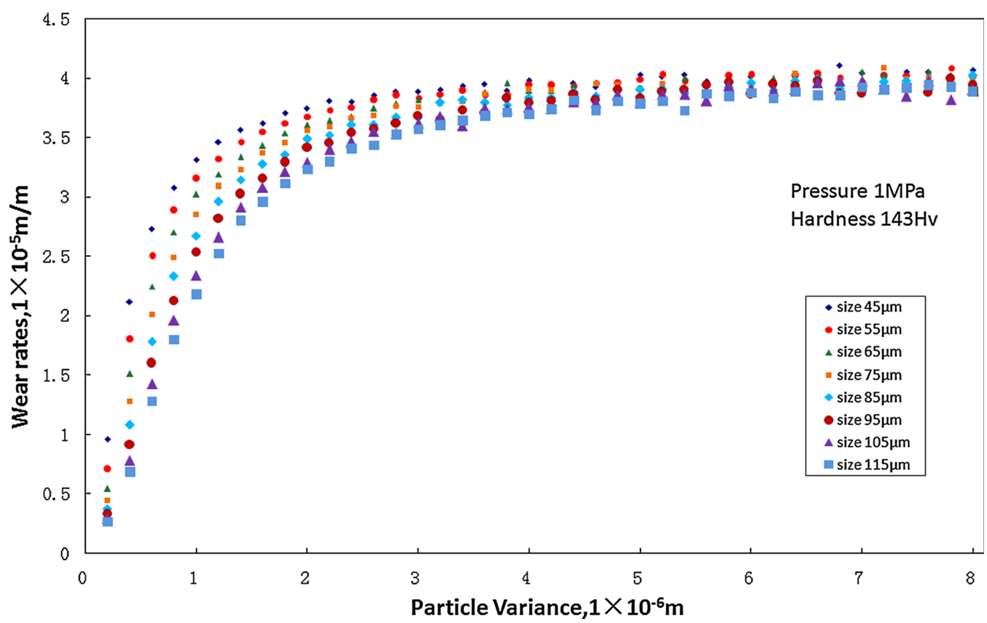

Fig. 7 The influence of particle size and variance on wear rate

of two different particle sizes, 80 and 40 μm, are shown in Fig. 8.

To further demonstrate the effect of variance of particle size on wear rates, variance values for different abrasive papers are marked in the curves of Fig. 9, related to relevant particle sizes. Clearly, wear rates increased when the average particle size and variance of abrasive particles increased. Considering the increase of wear rate caused by the increase of abrasive size with the same variance and by increase of variance with the same particle size (see Fig. 7), the variance of particle played a predominant role in determining the wear rate caused by increasing the average size of abrasive particles. Thus, the reason of the phenomenon of "critical size effect" can actually be contributed to the "critical variance effect."

Table 1 Experimental parameters of abrasive paper

Grit no.	80#	180#	220#	W40	W20
Size (μm)	200	80	68	40	20
Sq (μm)	35.6	17.1	15.7	7.58	5.01

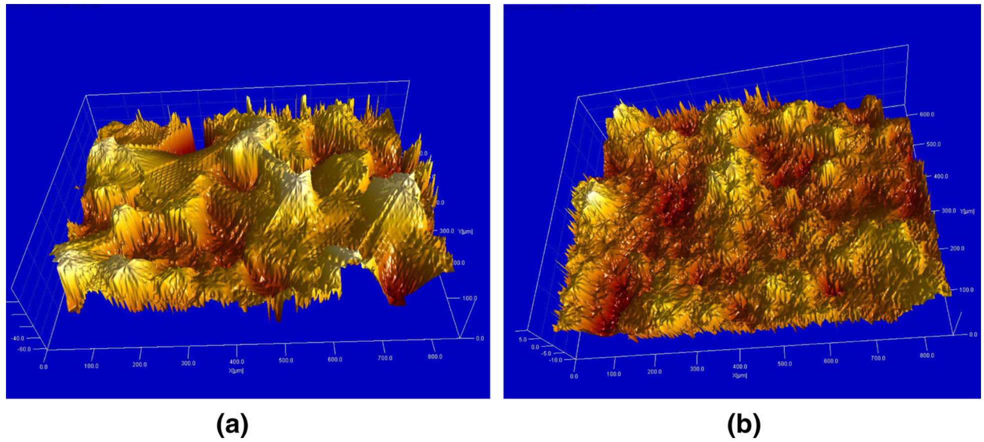

Fig. 8 Surface morphology of abrasive paper with different particle sizes. **a** average particle size 80 µm, **b** average particle size 40 µm

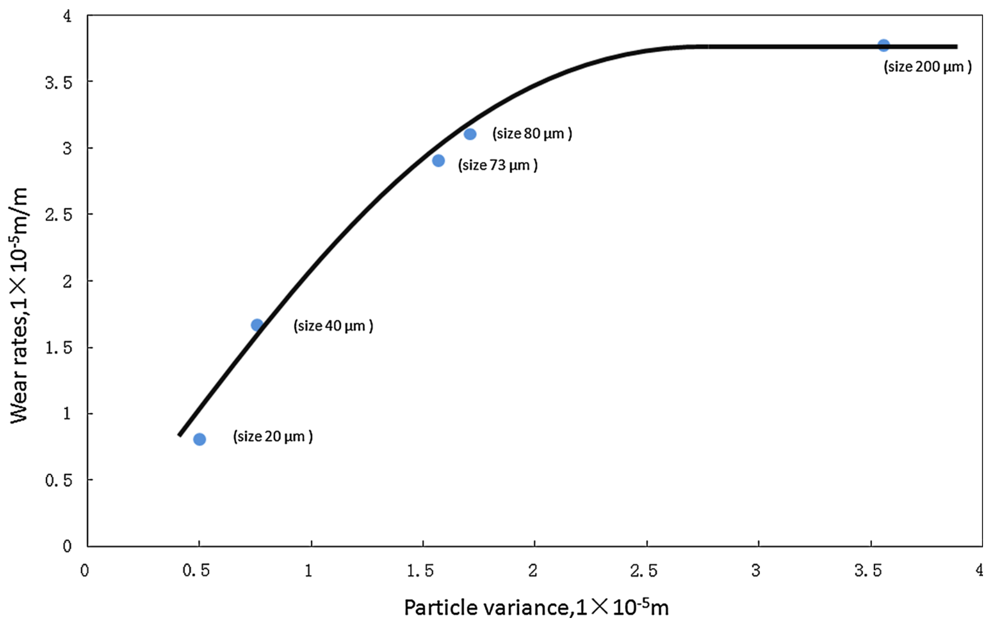

Fig. 9 Experimental results of the size effect on wear rates

In experiments, as large variance always comes together with large particle size, the influence of variance was rarely investigated in the past. Admittedly, the value of critical variance calculated from simulation (i.e., around 5 µm) was smaller than the value obtained from experiments in Fig. 9 (around 25 µm). We proposed the reason as below: when preparing the abrasive paper for abrasive wear test, all the abrasive particles were glued on the paper by resin. We measure the variance of abrasive particle by microX-AM optical profilometer. The resin is likely to be transparent under optical profilometer. It can be carefully observed from Fig. 8 that there are some sharp peaks because of reflection and scattering noise by the "transparent" resin, which results in a large variance experimental values. To this end, the critical variance of the real abrasive particles got from experiments turns to be larger than the critical variance calculated by our simulation. However, it is still important to notice that variance of particle size should be evaluated as another monitor to evaluate abrasive wear.

5 Conclusions

In this paper, we presented the study of the influence of variance of abrasive particle size by Monte Carlo simulation of two-body abrasion process. Surprisingly, variance of particle size was found to be another monitor to evaluate abrasive wear. Numbers of active particles decreased with increasing variances. The abrasive wear rates of material with the same particle average size also exhibited a rapid increase with increasing particle variance in a zone where the variance value is comparatively small. Furthermore, we demonstrated that it was the variance of particle size which played a crucial role in the "size effect" in abrasive wear. Our results showed that variance of abrasive particle size should be an important parameter to be considered in abrasive wear, providing useful insights in practical applications of abrasive wear.

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